On three domination-based identification problems on block graphs*

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— joint work with

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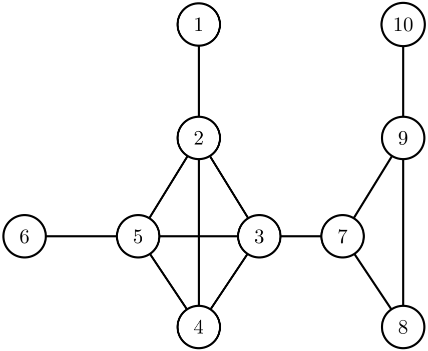
‡CNRS, LIRIS, Université Claude Bernard Lyon 1, France

Identifying code (ID-Code)
[Karpovsky et. al., 1998]

Locating-dominating set
(LD-set) [Slater, 1987]

Open-locating-dominating set
(OLD-set) [Seo & Slater, 2010]



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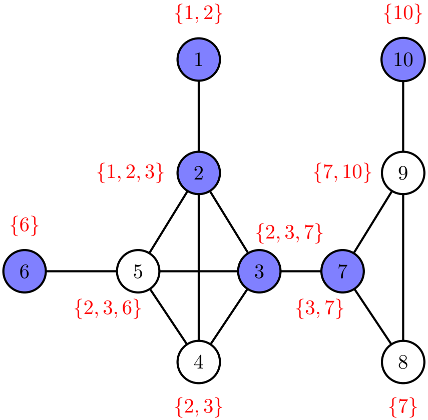

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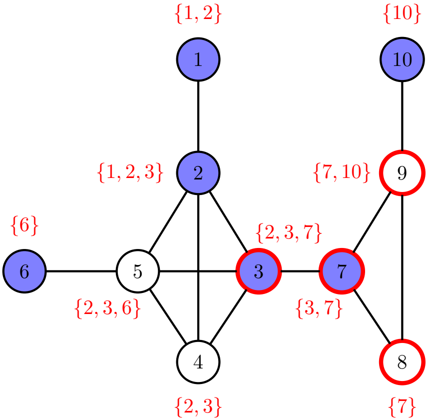

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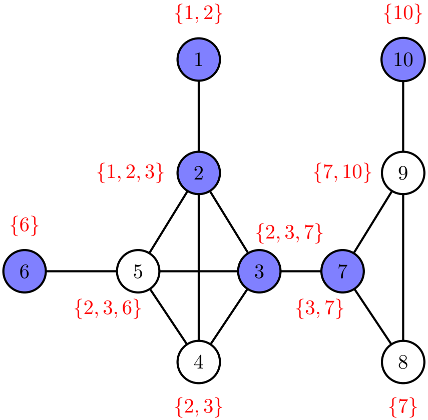

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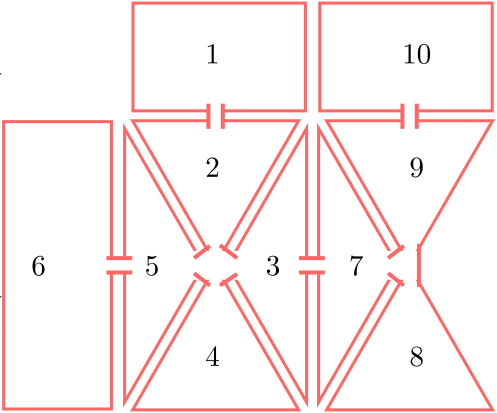

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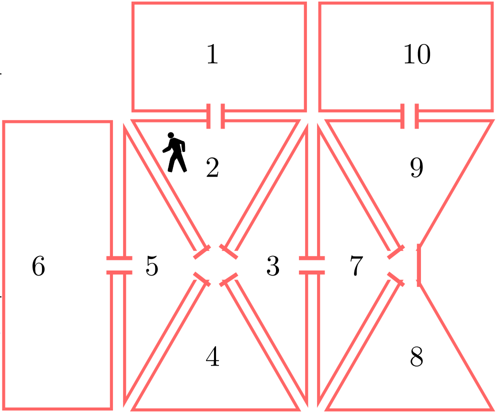

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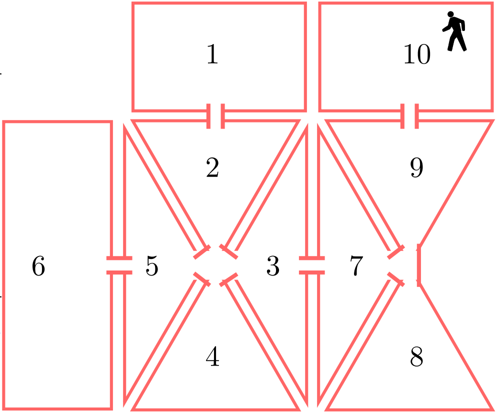

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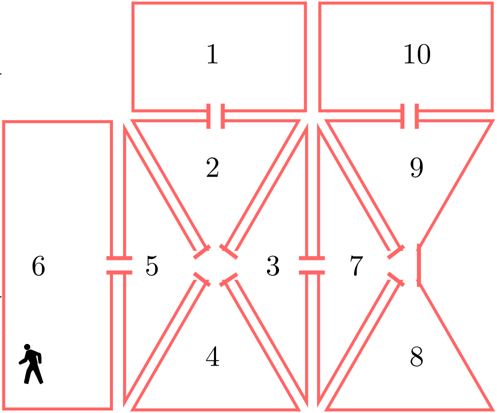

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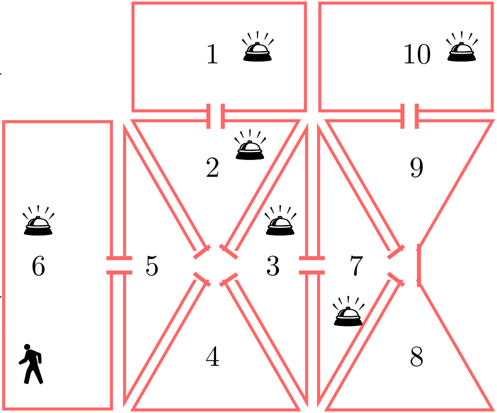

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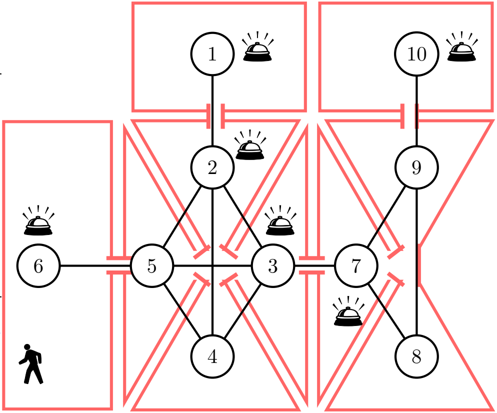

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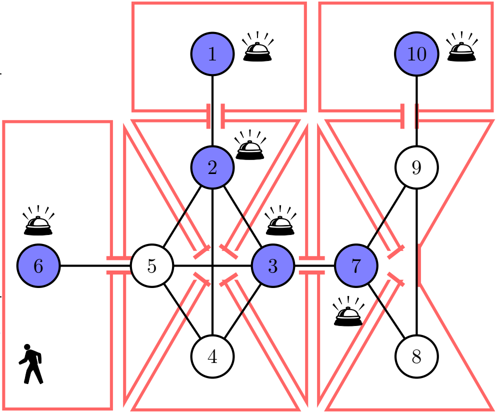

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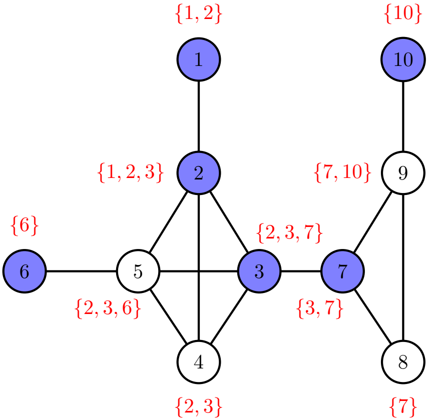

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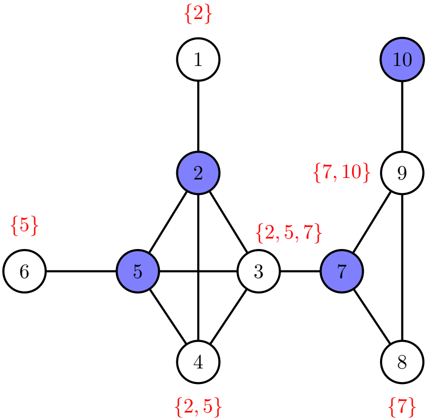

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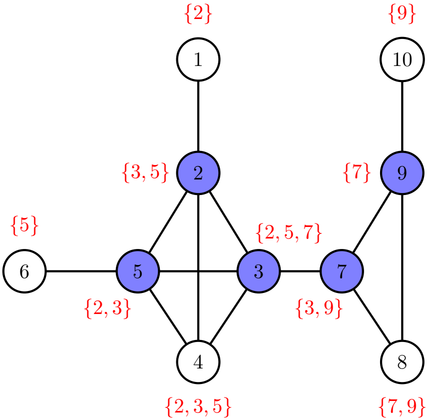

Identifying code (ID-Code) [Karpovsky et. al., 1998]

- B is a dominating set of G; Unique $N[v] \cap B$ $\forall v \in V(G)$.
- ID-number $\gamma^{ID}(G) = \min |B|$ $\forall$ ID-code B of G

Locating-dominating set (LD-set) [Slater, 1987]

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- LD-number $\gamma^{LD}(G) = \min |B|$ $\forall$ LD-set B of G

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- OLD-number $\gamma^{OLD}(G) = \min$ $|B|$ $\forall$ OLD-set B of G

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Exists only if G is closed twin-free.

Locating-dominating set (LD-set) [Slater, 1987]

- B is a dominating set of G; Unique $N(v) \cap B$ $\forall v \notin B$.
- LD-number $\gamma^{LD}(G) = \min |B|$ $\forall$ LD-set B of G

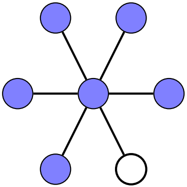
Always exists!

Open-locating-dominating set (OLD-set) [Seo & Slater, 2010]

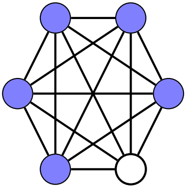
- B is total-dominating set of G; Unique $N(v) \cap B$ $\forall v \in V(G)$.
- OLD-number $\gamma^{OLD}(G) = \min |B|$ $\forall$ OLD-set B of G

Exists only if G is open twin-free and has no isolated vertices.

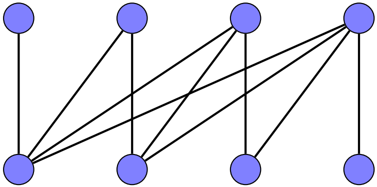
Some examples of code numbers



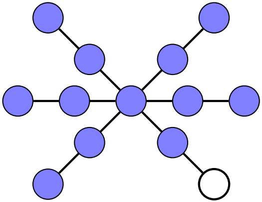
(a) $\gamma^{ID}(St_6) = \gamma^{LD}(St_6) = 5$



(b) $\gamma^{LD}(K_6) = \gamma^{OLD}(K_6) = 5$

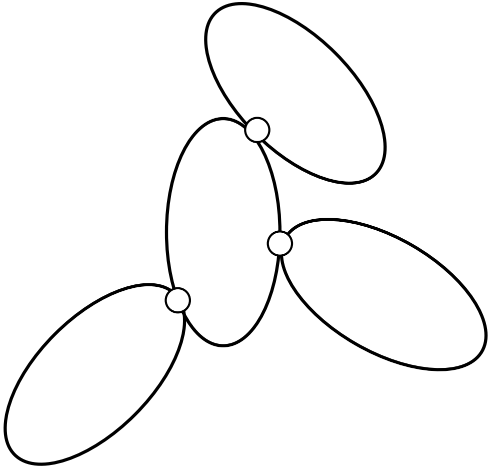


(a) $\gamma^{OLD}(HG) = 8$



(b) $\gamma^{OLD}(SSt_6) = 12$

Block graph



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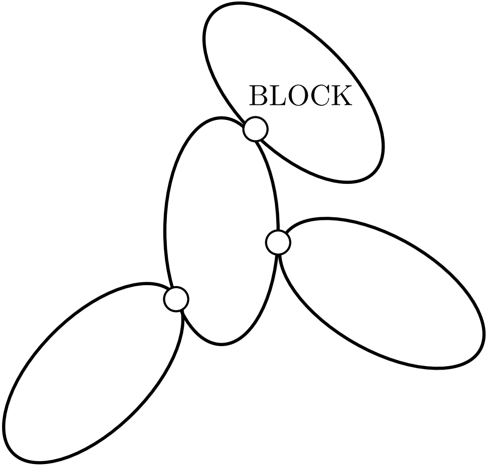

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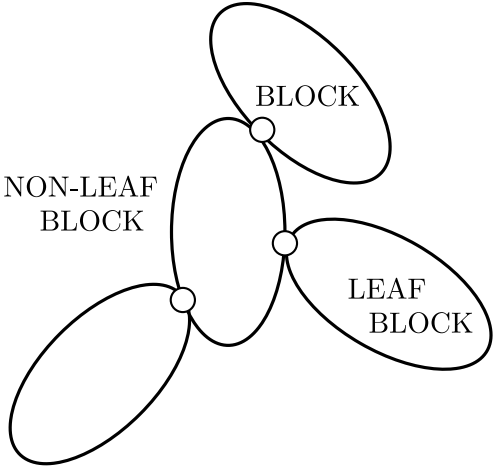

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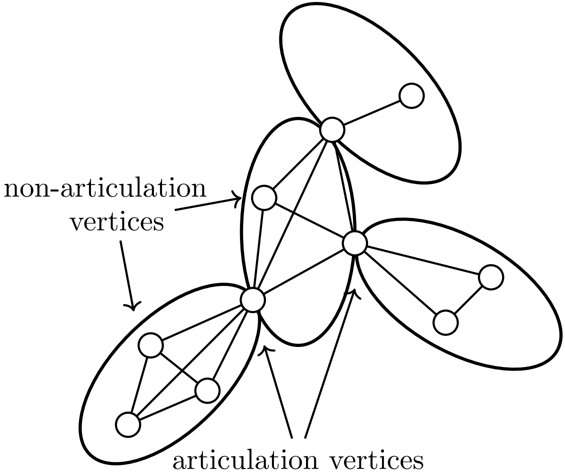
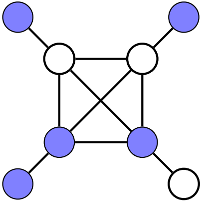


Results

Theorem (Conjecture. Argiroffo et. al. (2018))

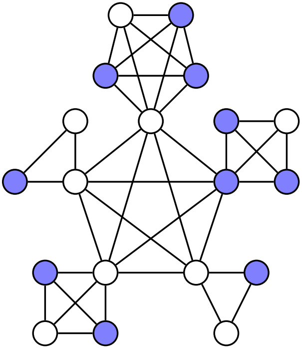
Let G be a closed twin-free block graph. Then $\gamma^{ID}(G) \leq n_Q(G)$, where $n_Q(G)$ is the number of blocks of G.



Results

Theorem

Let G be a block graph, $n_Q(G)$ be the number of blocks of G and $\mathcal{S} = \{S \subset V(G) : S$ is a maximal set of pairwise closed twins in some block $\}$. Then, $\gamma^{LD}(G) \leq n_Q(G) + \sum_{S \in \mathcal{S}}(|S| - 2)$.

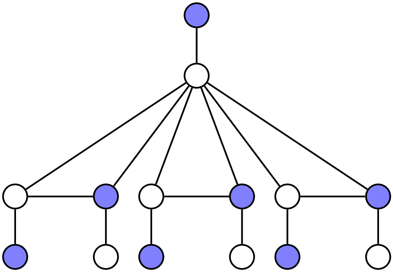


Results

Theorem

Let G be a twin-free block graph without isolated vertices. Then, $\gamma^{LD}(G) \leq \frac{1}{2}|V(G)|$.

Conjecture. Garijo et. al (2014): Let G be a twin-free graph without isolated vertices. Then, $\gamma^{LD}(G) \leq \frac{1}{2}|V(G)|$.

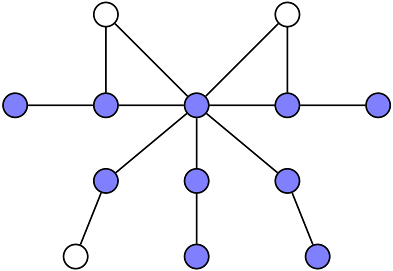


Results

Theorem

Let G be an open twin-free block graph, with no isolated vertices and $G \ncong P_2,\ P_4$. Let $m_Q(G)$ be the number of non-leaf blocks with at least one non-articulation vertex. Then, $\gamma^{OLD}(G) \leq |V(G)| - 1 - m_Q(G)$.

Foucaud et. al. (2021): For an open twin-free graph G, $\gamma^{OLD}(G) \leq |V(G)| - 1$ unless G is a half-graph (a special kind of bipartite graph)



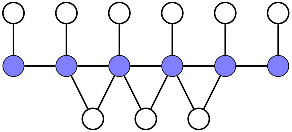
Results

Theorem

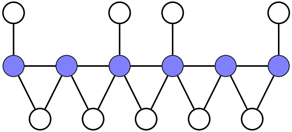
Let G be a block graph. Then

- $\gamma^{ID}(G) \geq \frac{|V(G)|}{3} + 1$,
- $\gamma^{OLD}(G) \geq \frac{|V(G)|}{3} + 1$ (except when $G \cong$ kite), and
- $\gamma^{LD}(G) \geq \frac{|V(G)|+1}{3}$.

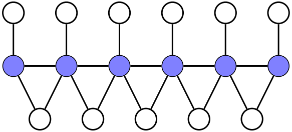
General lower bound: $\gamma^{ID}(G), \gamma^{LD}(G), \gamma^{OLD}(G) \geq \lceil \log_2(|V(G)| + 1) \rceil$.



(a) $\gamma^{ID}(G) = 6$, $|V(G)| = 15$



(b) $\gamma^{OLD}(G) = 6$, $|V(G)| = 15$



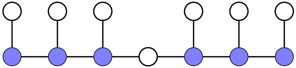
(c) $\gamma^{LD}(G) = 6$, $|V(G)| = 17$

Results

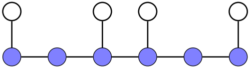
Theorem

Let G be a block graph. Then

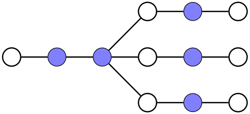
- $\gamma^{ID}(G) \geq \frac{3(n_Q(G)+2)}{7}$,
- $\gamma^{LD}(G) \geq \frac{n_Q(G)+2}{3}$, and
- $\gamma^{OLD}(G) \geq \frac{n_Q(G)+3}{2}$.



(a) $\gamma^{ID}(G) = 6$, $|V(G)| = 13$



(b) $\gamma^{OLD}(G) = 6$, $|V(G)| = 10$



(c) $\gamma^{LD}(G) = 5$, $|V(G)| = 12$

Thank you!